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Converting GPS Coordinates
$(\phi\lambda h)$ **to Navigation Coordinates**
(ENU)

S.P. Drake

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ABSTRACT

This technical note outlines a more efficient procedure for converting latitude, longitude and height in the GPS coordinates to local east, north, up coordinates in the navigation frame.

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EXECUTIVE SUMMARY

In many applications relevant to the Australian Defence Force (ADF) it is necessary to convert the Global Positioning System (GPS) coordinates of latitude, longitude and height to a local navigation frame with coordinates, east, north and up. For example when testing navigation instruments, such as the inertial navigation system (INS), it is often helpful to compare these measurements with those obtained from an independent GPS receiver. An INS records the east, north and up displacement from its point of origin. To compare INS and GPS measurements we need to transform GPS coordinates to navigation coordinates. Furthermore, east, north, up coordinates are essential in determining the line of sight for terrain data given as latitude, longitude and height, such as digital terrain elevation data (DTED). For large amounts of data, e.g, trial data, this process may be very computationally intensive.

Means for converting GPS data to navigation frame coordinates already exist. However, the method presented in this report is roughly three times faster than a commonly employed one. The coordinate transformation routine outlined in this report is accurate to within 10m over a range of 60km.

The coordinate transformation method outlined here will be used in the Navwar simulation package currently under development in DSTO Edinburgh. It will have the effect of significantly reducing the processing time of line of sight calculations in jamming models.

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Glossary

ADF Australian Defence Force

DSTO Defence Science Technology Organisation

GPS Global Positioning System

ECEF Earth Centered Earth Fixed

ENU East North Up

INS Inertial Navigation System

RMS Root Mean Square

WGS84 World Geodetic System 1984

1   Introduction

The aim of this report is to provide a useful and efficient means of using Global Positioning System (GPS) data to determine local range and bearing measurements. This is particularly pertinent owing to the ubiquity of GPS receivers.

The output of a GPS receiver is latitude, longitude and height in the World Geodetic System 1984 (WGS84) coordinate frame. However in many applications we are interested in the range and bearing of these coordinates with respect to a reference point, for example the range and bearing of an aeroplane, with respect to the control tower. When testing other navigation instruments, such as the inertial navigation system (INS), it is often helpful to compare these measurements with those obtained from an independent GPS receiver. An INS records the east, north and up displacement from its point of origin. To compare INS and GPS measurements we need to transform WGS84 coordinates to navigation coordinates. Furthermore, (east, north, up) coordinates are essential in determining the line of sight for terrain data given as latitude, longitude and height, such as digital terrain elevation data (DTED).

At this point the reader may ask, "hasn't this already been done?" Yes, it has been done before, for example GPSoft$^{\text{TM}}$ have Matlab$^{\text{TM}}$ toolboxes which convert GPS coordinates into local navigation coordinates. In this report we derive a method which does this conversion three times faster than GPSoft$^{\text{TM}}$. This difference is significant when dealing with large amounts of data.

2   Converting from WGS84 to navigation coordinates

Navigation coordinates are determined by the fitting of a tangent plane to a fixed point on the surface of the Earth. The $\hat{e}$ axis points east, the $\hat{n}$ axis north, and $\hat{u}$ axis points perpendicular to the tangent plane and away from the centre of the Earth.
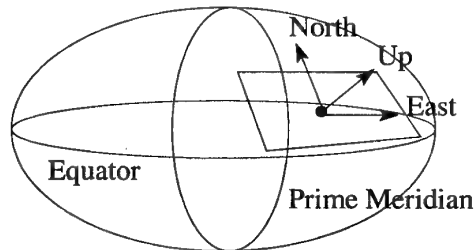

Figure 1: Navigation and ECEF frames.

What follows below is a derivation of the transformation from latitude, longitude and height ($\phi\lambda h$) to east, north and up (ENU). It has been included for completeness but the reader may safely jump to equation (4) if they are only interested in the final result.

The transformation from $\phi\lambda h$ to ENU is a three stage process:

1. Determine latitude, longitude and height of reference point, e.g. location of radar. In this report it is the arbitrary point (ϕ, λ, h).

2. Express small changes in latitude, longitude and height in Earth Centred Earth Fixed (ECEF) coordinates.
   GPS coordinates can be converted into ECEF coordinates using the following formulae:

$$
\begin{aligned}
x &= \left(\frac{a}{\chi} + h\right)\cos\phi\cos\lambda \\
y &= \left(\frac{a}{\chi} + h\right)\cos\phi\sin\lambda \\
z &= \left(\frac{a(1-e^2)}{\chi} + h\right)\sin\phi
\end{aligned}
\tag{1}
$$

Where

$$
\chi = \sqrt{1 - e^2\sin^2\phi},
$$

a and e^2 are the semi-major axis and the first numerical eccentricity of the Earth respectively.

To convert small changes in latitude, longitude and height into ECEF coordinates we need to Taylor expand equation (1) about $\phi \to \phi + d\phi$, $\lambda \to \lambda + d\lambda$, and $h \to h + dh$.

$$
\begin{aligned}
dx &= \left(\frac{-a\cos\lambda\sin\phi(1-e^2)}{\chi^3} - h\cos\lambda\sin\phi\right)d\phi - \left(\frac{a\sin\lambda\cos\phi}{\chi} + h\sin\lambda\cos\phi\right)d\lambda \\
&+ \cos\phi\cos\lambda dh + \left(\frac{1}{4}a\cos\phi\cos\lambda\left(-2 - 7e^2 + 9e^2\cos^2\phi\right) - \frac{1}{2}h\cos\lambda\cos\phi\right)d\phi^2 \\
&+ \left(\frac{a\sin\lambda\sin\phi(1-e^2)}{\chi^3} + h\sin\lambda\sin\phi\right)d\phi d\lambda - \cos\lambda\sin\phi dh d\phi \\
&+ \left(\frac{-a\cos\lambda\cos\phi}{2\chi} - \frac{1}{2}h\cos\lambda\cos\phi\right)d\lambda^2 - \sin\lambda\cos\phi dh d\lambda + \mathcal{O}(d\theta^3) + \mathcal{O}(dh d\theta^2) \\
dy &= \left(\frac{-a\sin\lambda\sin\phi(1-e^2)}{\chi^3} - h\sin\lambda\sin\phi\right)d\phi + \left(\frac{a\cos\lambda\cos\phi}{\chi} + h\cos\lambda\cos\phi\right)d\lambda \\
&+ \sin\phi\cos\lambda dh + \left(\frac{1}{4}a\cos\phi\sin\lambda\left(-2 - 7e^2 + 9e^2\cos^2\phi\right) - \frac{1}{2}h\sin\lambda\cos\phi\right)d\phi^2 \\
&+ \left(\frac{-a\cos\lambda\sin\phi(1-e^2)}{\chi^3} - h\cos\lambda\sin\phi\right)d\phi d\lambda - \sin\lambda\sin\phi dh d\phi \\
&+ \left(\frac{-a\sin\lambda\cos\phi}{2\chi} - \frac{1}{2}h\sin\lambda\cos\phi\right)d\lambda^2 + \cos\lambda\cos\phi dh d\lambda + \mathcal{O}(d\theta^3) + \mathcal{O}(dh d\theta^2) \\
dz &= \left(\frac{a(1-e^2)\cos\phi}{\chi^3} + h\cos\phi)\right)d\phi + \sin\phi dh + \cos\phi dh d\phi \\
&+ \left(\frac{1}{4}a\sin\phi\left(-2 - e^2 + 9e^2\cos^2\phi\right) - \frac{1}{2}h\sin\phi\right)d\phi^2 + \mathcal{O}(dh d\phi^2),
\end{aligned}
\tag{2}
$$

where $d\theta$ is either $d\phi$ or $d\lambda$.

3. By means of a rotation, displacements in ECEF coordinates are transformed to ENU coordinates.
   The ECEF coordinates (dx, dy, dz) are orientated in such a way that from the centre of the Earth the $\hat{z}$ points in the direction of true north, $\hat{x}$ points in the direction of the prime meridian and the direction of $\hat{y}$ is 90° from the prime meridian, see figure 1. The orientation of ENU coordinates is determined by rotating the ECEF coordinates; firstly about the $\hat{z}$ axis by λ degrees and then the new $\hat{y}$ axis by ϕ

degrees,

$$
\begin{pmatrix} de \\ dn \\ du \end{pmatrix} = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}. \tag{3}
$$

Substituting equation (2) into equation (3), and ignoring terms of $\mathcal{O}(d\theta^3)$ and $\mathcal{O}(dhd\theta^2)$ and higher, we get

$$
\begin{aligned}
de &= \left(\tfrac{a}{\chi}+h\right)\cos\phi\,d\lambda - \left(\tfrac{a(1-e^2)}{\chi^3}+h\right)\sin\phi\,d\phi\,d\lambda + \cos\phi\,d\lambda\,dh \\
dn &= \left(\tfrac{a(1-e^2)}{\chi^3}+h\right)d\phi + \tfrac{3}{2}a\cos\phi\sin\phi e^2 d\phi^2 + dh\,d\phi \\
&\quad + \tfrac{1}{2}\sin\phi\cos\phi\left(\tfrac{a}{\chi}+h\right)d\lambda^2 \\
du &= dh - \tfrac{1}{2}a\left(1-\tfrac{3}{2}e^2\cos\phi+\tfrac{1}{2}e^2+\tfrac{h}{a}\right)d\phi^2 \\
&\quad - \tfrac{1}{2}\left(\tfrac{a\cos^2\phi}{\chi}-h\cos^2\phi\right)d\lambda^2 \quad .
\end{aligned} \tag{4}
$$

Note:

The coordinates (ϕ,λ,h) are ellipsoidal (WGS84), not spheroidal (geocentric). The difference is most easily explained by figure 2.
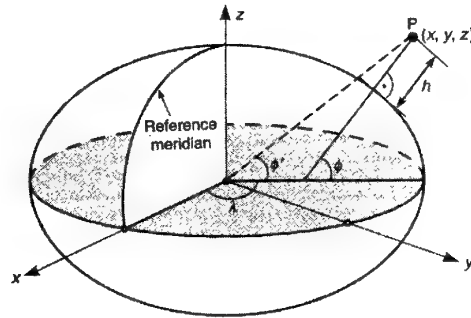
Figure 2: Ellipsoidal and spheroidal coordinates

3 Matlab Program: dllh2denu

Armed with equation (4) we are now in a position to write a Matlab program. The code for our program *dllh2denu.m* is given in appendix A. The routine *dllh2denu* is simple to use, it requires two inputs; the first is a vector indicating the location of the reference point (e.g. radar) in latitude, longitude in degrees and height in metres. The first input is a 1×3 vector of the form

$$\text{llh0} = [\phi, \lambda, h].$$

3

The second input is a $n \times 3$ matrix indicating the location of the points of interest (e.g. the location of the aeroplane). The second input is of the form

$$\text{llh} = \begin{bmatrix} \phi_1 & \lambda_1 & h_1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \phi_n & \lambda_n & h_n \end{bmatrix}.$$

The resulting output is a $n \times 3$ matrix with the first, second, and third columns, representing the east, north and up displacement respectively in metres

$$\text{denu} = \begin{bmatrix} e_1 & n_1 & u_1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ e_n & n_n & u_n \end{bmatrix}.$$

As an example suppose the location of the reference point is

$$[\phi = 39°, \lambda = -132°, h = 0]$$

and three points of interested are located at

ϕ	λ	h
$39° + 0.5°$	$-132°$	0
$39° + 0.5°$	$-132° + 0.5°$	0
$39° + 0.5°$	$-132° + 0.5°$	60000

To determine the location of these points in ENU coordinates we would run *dllh2denu* in the following way:

```
>> format short g
>> llh0 = [39, -132, 0]

llh0 =

    39   -132      0

>> llh = [39 + 0.5, -132, 0;
39 + 0.5, -132+0.5, 0;
39 + 0.5, -132+0.5, 60000]

llh =

        39.5            -132               0
        39.5          -131.5               0
        39.5          -131.5           60000
```

```
>> denu = dllh2denu(llh0,llh)

denu =

        0       55510      -242.2
    43008       55629     -389.07
    43415       56153       59611
```

4 Accuracy of Method

The displacements as determined by Eq. (4) are approximations, since the Taylor expansion of Eq. (1) was truncated at second order. To estimate the error of this approximation it suffices to take an upper estimate on the effect of third order terms $\mathcal{O}(d\theta^3)$ and $\mathcal{O}(dh d\theta^2)$, where $d\theta$ is either $d\phi$ or $d\lambda$. The greatest possible third order effect is at the equator. Third order terms at this point are of the order $a d\theta^3$ and $dh d\theta^2$. If we limit $d\theta$ to 0.5°and dh to 60km then this effect is less than 7m. Near the equator an angular offset of latitude or longitude of 0.05° equates to roughly 60km. Hence as a rule of thumb

> Equation (4), and the hence the program in appendix A, may be used if an error of less than 10m is acceptable and the range of measurements is less then 60km.

Should the range exceed 60km, or an accuracy much less than 10m be required, it is necessary to calculate dx, dy, dz using the method employed by GPSoft^{TM}. This method firstly converts each point ϕ_i, λ_i, h_i to x_i, y_i, z_i using Eq. (1). The displacements in ENU coordinates are then calculated by using Eq. (3), and the fact that $dx_i = x_i - x_0$, where x_0 is the location of the reference point in ECEF coordinates.

To validate our code let us compare ENU displacements as determined by GPSoft^{TM}'s *llh2xyz.m* and *xyz2enu.m* (ENU_{GPSoft}) with those determined by *dllh2denu.m* (ENU_{sam}). Taking the same reference and data points as before we find that

ϕ, λ, h	ENU_{GPSoft}	ENU_{sam}	RMS difference
39°, −132°, 0	[0, 0,0]	[0, 0, 0]	0m
39+0.5°, -132°, 0	[0, 55509.42,−242.21]	[0, 55510.13,−242.20]	0.7m
39+0.5°, −132 + 0.5°, 0	[43006.16, 55627.52,−388.04]	[43008.36, 55629.06,−389.07]	2.88m
39+0.5°, −132 + 0.5°, 60000	[43410.18, 56152.22,59608.30]	[43415.27, 56152.66,59610.93]	5.75m

5 Efficiency of Method

As already mentioned in the introduction, the main motivation behind this work is to speed up the processing of data. In this section a simple time comparison experiment

shows that data can be processed roughly three times faster using the method outlined in this report.

To perform this comparison fairly the programs *llh2xyz.m* and *xyz2enu.m* had to be modified so that they could process more than one coordinate at a time. The program *comparison.m* was used to compare the CPU time taken to calculate ENU using these two methods, the code is in appendix A. The results are shown below.

```
>> comparison
t_sat = 0.935 +- 0.01354
t_sam = 0.344 +- 0.005164
```

We can see from these results that the method presented in this report is roughly three times faster than that used by GPSoft$^{\text{TM}}$.

6   Conclusion

The aim of this note is to report work in DSTO to establish an efficient method by which to transform GPS coordinates to local navigation coordinates east, north and up. To this end equation (4) was derived and a Matlab program *dllh2denu.m* was created to evaluate it for any number of data points.

Equation (4), and consequently the program *dllh2denu.m*, are valid only if

a) The range is less than 60km, and

b) an error of a few meters is acceptable.

The accuracy and efficiency was compared with the method of GPSoft$^{\text{TM}}$and a simple test showed that they agreed to within a metre, but our method was three times faster than theirs.

Appendix A: Matlab Code

A.1 dllh2denu.m

```
function denu = dllh2denu(llh0,llh)

%%%%%%%%%%%CONSTANTS
a = 6378137;
b = 6356752.3142;
e2 = 1 - (b/a)^2;
%%%%%%%%%%%Location of reference point in radians
phi = llh0(1)*pi/180;
lam = llh0(2)*pi/180;
h   = llh0(3)
%%%%%%%%%%%Location of data points in radians
dphi= llh(:,1)*pi/180 - phi;
dlam= llh(:,2)*pi/180 - lam;
dh  = llh(:,3) - h;
%%%%%%%%%%%Some useful definitions
tmp1 = sqrt(1-e2*sin(phi)^2);
cl = cos(lam);
sl = sin(lam);
cp = cos(phi);
sp = sin(phi);
%%%%%%%%%%%Transformations
de = (a/tmp1+h)*cp*dlam - (a*(1-e2)/(tmp1^3)+h)*sp.*dphi.*dlam +cp.*dlam.*dh;

dn = (a*(1-e2)/tmp1^3 + h)*dphi + 1.5*cp*sp*a*e2*dphi.^2 + sp^2.*dh.*dphi + ...
        0.5*sp*cp*(a/tmp1 +h)*dlam.^2;

du = dh - 0.5*(a-1.5*a*e2*cp^2+0.5*a*e2+h)*dphi.^2 - ...
        0.5*cp^2*(a/tmp1 -h)*dlam.^2;

denu = [de, dn, du];
```

A.2   comparison.m

```
clear all
N=100000; % Number data points
loops = 10;
%%%%%%%%%%%%%%%%%%%Initialization
phi0 = 39;
lam0 = -132;
h0 = 0;
orgllh = [phi0,lam0,0];
phi = zeros(N,1);
lam = zeros(N,1);
h = zeros(N,1);
enu = zeros(N,3);
xyz = zeros(N,3);
denu_sat = zeros(N,3);
denu_sam = zeros(N,3);
t1 = zeros(loops,1);
t2 = zeros(loops,1);
%%%%%%%%%%%%%%%%%%%Coordinate data
for i = 1:N;
  phi(i) = phi0 + 0.5*i/N;
  lam(i) = lam0 + 0.5*i/N;
  h(i) = h0 +i;
end
llh = [phi, lam, h];
%%%%%%%%%%%%%%%%%% Comparison loop
for j = 1:loops
  t = cputime;
  orgxyz = llh2xyzmodi(orgllh);
  xyz = llh2xyzmodi(llh);
  denu_sat = xyz2enumodi(xyz,orgxyz);
  t1(j) = cputime -t;
  t = cputime;
  denu_sam = dllh2denu(orgllh,llh);
  t2(j) = cputime -t;
end
%%%%%%%%%%%%%%%%%% Statistics
t1m = mean(t1);
t1std = std(t1);
t2m = mean(t2);
t2std = std(t2);
disp(['t_sat = ',num2str(t1m), ' +- ', num2str(t1std)]);
disp(['t_sam = ',num2str(t2m), ' +- ', num2str(t2std)]);
```

Appendix B: Glossary, Physical Constants and Mathematical notation

Symbol	Value	Meaning
a	6378137	length of Earth's semi-major axis in metres
b	6356752.3142	length of Earth's semi-minor axis in metres
e^2	$6.69437999013 \times 10^{-3}$	first numerical eccentricity
ϕ		latitude
λ		longitude
h		height
e		displacement in the east direction of the navigation frame
n		displacement in the north direction of the navigation frame
u		up displacement in the navigation frame
x		displacement in the $\hat{x}$ direction of the ECEF frame
y		displacement in the $\hat{y}$ direction of the ECEF frame
z		displacement in the $\hat{z}$ direction of the ECEF frame

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S. P. Drake

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19. ABSTRACT
This technical note outlines a more efficient procedure for converting latitude, longitude and height in the GPS coordinates to local east, north, up coordinates in the navigation frame.